FLOW IN OPEN CHANNELS

- **A broad coverage of topics in open-channel flow has been selected for this chapter.**
- **In this chapter open-channel flow is first classified and then the shape of optimum canal cross sections is discussed, followed by a section on flow through a floodway. The hydraulic jump and its application to stilling basins is then treated, followed by a discussion of specific energy and critical depth which leads into transitions and then gradually varied flow.**
- **Water-surface profiles are classified and related to channel control sections. In conclusion positive and negative surge waves in a rectangular channel are analyzed, neglecting effects of friction.**
- **The presence of a free surface makes the mechanics of flow in open channels more complicated than closed-conduit flow. The hydraulic grade line coincides with the free surface, and, in general, its position is unknown.**

- **For laminar flow to occur, the cross section must be extremely small, the velocity very small, or the kinematic viscosity extremely high.**

- **One example of laminar flow is given by a thin film of liquid flowing down an inclined or vertical plane. Pipe flow has a lower critical Reynolds number of 2000, and this same value may be applied to an open channel when the diameter *D* is replaced by 4*R*, *R* is the hydraulic radius, defined as the cross-sectional flow area of the channel divided by the wetted perimeter.**

- **In the range of Reynolds number, based on *R* in place of *D*, R = *VR*/*v* < 500 flow is laminar, 500 < R < 2000 flow is transitional and may be either laminar or turbulent, and R > 2000 flow is generally turbulent.**

- **Most open-channel flows are turbulent, usually with water as the liquid. The methods for analyzing open-channel flow are not developed to the extent of those for closed conduits. The equations in use assume complete turbulence, with the head loss proportional to the square of the velocity.**

CLASSIFICATION OF FLOW

- **Open-channel flow occurs in a large variety of forms, from flow of water over the surface of a plowed field during a hard rain to the flow at constant depth through a large prismatic channel. It may be classified as steady or unsteady, uniform or nonuniform.**

- ***Steady uniform flow* occurs in very long inclined channels of constant cross section in those regions where terminal velocity has been reached, i.e., where the head loss due to turbulent flow is exactly supplied by the reduction in potential energy due to the uniform decrease in elevation of the bottom of the channel.**

- **The depth for steady uniform flow is called the normal depth. In steady uniform flow the discharge is constant and the depth is everywhere constant along the length of the channel.**

- *Steady nonuniform flow* occurs in any irregular channel in which the discharge does not change with the time; it also occurs in regular channels when the flow depth and hence the average velocity change from one cross section to another.

- For gradual changes in depth or section, called gradually varied flow, methods are available, by numerical integration or step-by-step means, for computing flow depths for known discharge, channel dimensions and roughness, and given conditions at one cross section.

- *Unsteady uniform flow* rarely occurs in open-channel flow. *Unsteady nonuniform flow* is common but difficult to analyze. Wave motion is an example of this type of flow, and its analysis is complex when friction is taken into account.

- **Flow is also classified as *tranquil* or *rapid*. When flow occurs at low velocities so that a small disturbance can travel upstream and thus change upstream conditions, it is said to be tranquil flow (the Froude number F < 1). Conditions upstream are affected by downstream conditions, and the flow is controlled by the downstream conditions.**

- **When flow occurs at such high velocities that a small disturbance, such as an elementary wave is swept downstream, the flow is described as shooting or rapid (F > 1). Small changes in downstream conditions do not effect any change in upstream conditions; hence, the flow is controlled by upstream conditions.**

- **When flow is such that its velocity is just equal to the velocity of an elementary wave, the flow is said to be critical (F = 1).**

- **The terms "subcritical" and "supercritical" are also used to classify flow velocities. *Subcritical* refers to tranquil flow at velocities less than critical, and *supercritical* corresponds to rapid flows when velocities are greater than critical.**

Velocity Distribution

- The velocity at a solid boundary must be zero, and in open-channel flow it generally increases with distance from the boundaries.

- The maximum velocity does not occur at the free surface but is usually below the free surface a distance of 0.05 to 0.25 of the depth.

- The average velocity along a vertical line is sometimes determined by measuring the velocity at 0.6 of the depth, but a more reliable method is to take the average of the velocities at 0.2 and 0.8 of the depth, according to measurements of the U.S. Geological Survey.

BEST HYDRAULUIC CHANNEL CROSS SECTIONS

- Some channel cross sections are more efficient than others in that they provide more area for a given wetted perimeter.

- From the Manning formula it is shown that when the area of cross section is a minimum, the wetted perimeter is also a minimum, and so both lining and excavation approach their minimum value for the same dimensions of channel.

- The *best hydraulic section* is one that has the least wetted perimeter or its equivalent, the least area for the type of section.
- The Manning formula is

$$\boxed{Q = \frac{1}{n} A R^{2/3} S^{1/2}} \qquad (12.2.1)$$

- in which Q is the discharge (L^3/T), A the cross-sectional flow area, R (area divided by wetted perimeter P) the hydraulic radius, S the slope of energy grade line, n the Manning roughness factor, C_m an empirical constant ($L^{1/3}/T$) equal to 1.49 in USC units and to 1.0 in SI units.

- With Q, n, and S known, Eq. (12.2.1) can be written

$$\boxed{A = cP^{2/5}} \tag{12.2.2}$$

- in which c is known. This equation shows that P is a minimum when A is a minimum.
- To find the best hydraulic section for a *rectangular* channel (Fig. 12.1) $P = b + 2y$ and $A = by$. Then

$$\boxed{A = (P - 2y)y = cP^{2/5}}$$

- Differentiating with respect to y gives

$$\boxed{\left(\frac{dP}{dy} - 2\right)y + P - 2y = \frac{2}{5}cP^{-3/5}\frac{dP}{dy}}$$

- Setting $dP/dy = 0$ gives $P = 4y$, or since $P = b + 2y$,

$$\boxed{b = 2y} \tag{12.2.3}$$

- Therefore, the depth is one-half the bottom width, independent of the size of rectangular section.

- To find the best hydraulic *trapezoidal* section (Fig. 12.2) $A = by + my^2$, $P = b + 2y\sqrt{(1 + m^2)}$. After eliminating b and A in these Eq`ns and Eq. (12.2.2),

$$A = by + my^2 = (P - 2y\sqrt{1+m^2})y + my^2 = cP^{2/5} \qquad (12.2.4)$$

- By holding *m* constant and by differentiating with respect to *y*, $\partial P/\partial y$ is set equal to zero; thus

$$P = 4y\sqrt{1+m^2} - 2my \qquad (12.2.5)$$

- Eq. (12.2.4) is differentiated with respect to *m*, and $\partial P/\partial m$ is set equal to zero, producing

$$\frac{2m}{\sqrt{1+m^2}} = 1 \qquad m = \frac{\sqrt{3}}{3}$$

- And after substituting for *m* in Eq. (12.2.5),

$$P = 2\sqrt{3}\,y \qquad b = 2\frac{\sqrt{3}}{3}y \qquad A = \sqrt{3}\,y^2 \qquad (12.2.6)$$

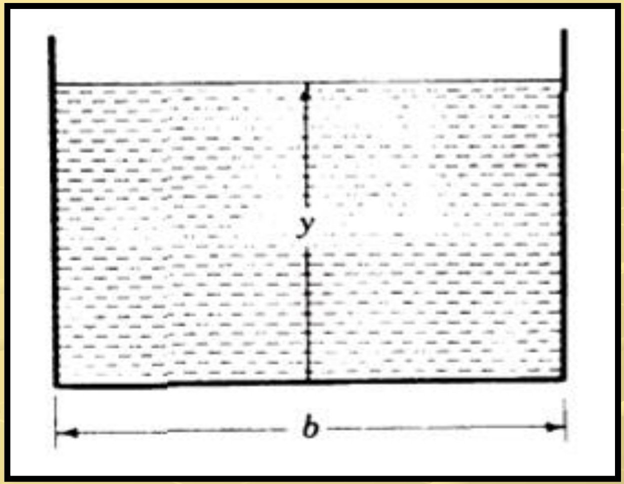
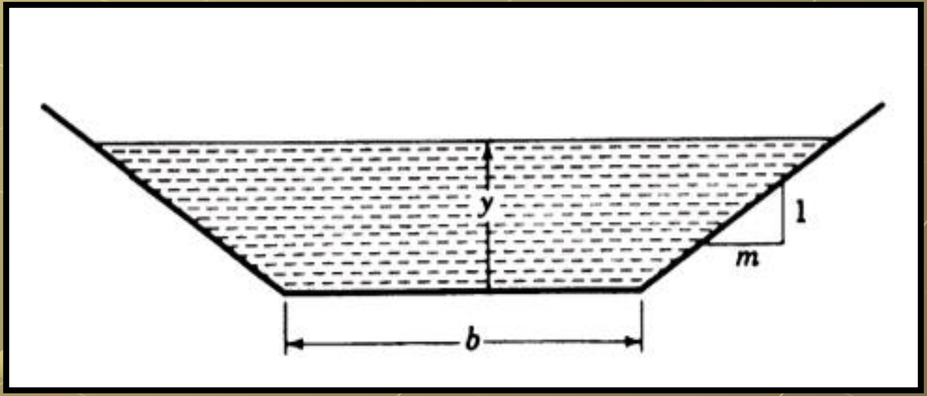


Figure 1 **Rectangular cross section**

Figure 2
Trapezoidal cross section



Example 1

- **Determine the dimensions of the most economical trapezoidal brick-lined channel to carry 200 m^3/s with a slope of 0.0004.**

Solution

- **With Eq. (12.2.6),**

$$\boxed{R = \frac{A}{P} = \frac{y}{2}}$$

- **and by substituting into Eq. (12.2.1)**

$$\boxed{200 = \frac{1.00}{0.016}\sqrt{3}\,y^2\left(\frac{y}{2}\right)^{2/3}\sqrt{0.0004}}$$

- **or**

$$\boxed{y^{8/3} = 146.64} \qquad \boxed{y = 6.492 \text{ m}}$$

- **and from Eq. (12.2.6) *b* = 7.5 m.**

STEADY UNIFORM FLOW IN A FLOODWAY

- A practical open-channel problem of importance is the computation of discharge through a floodway (Fig. 12.3). In general, the floodway is much rougher than the river channel and its depth (and hydraulic radius) is much less. The slope of energy grade line must be the same for both portions.

- The discharge for each portion is determined separately, using the dashed line of Fig. 12.3 as the separation line for the two sections (but not as solid boundary), and then the discharges are added to determine the total capacity of the system.

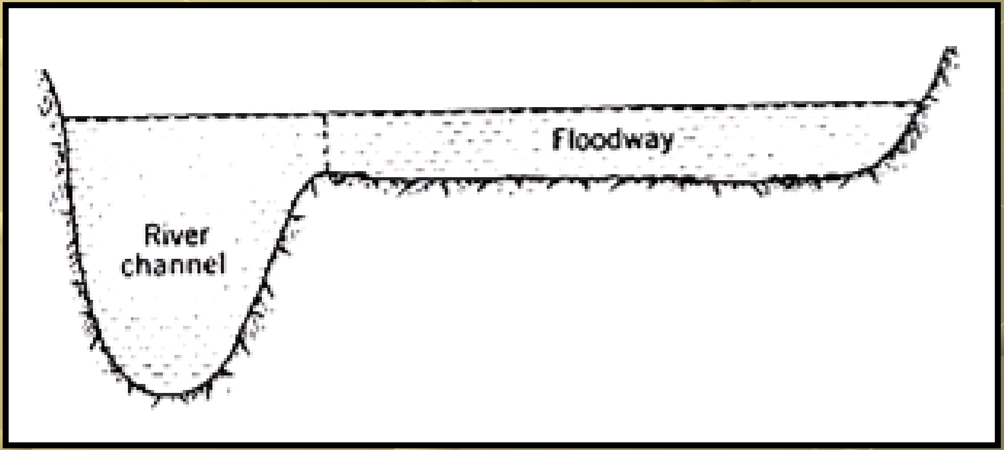


Figure 3 Floodway cross section

- Since both portions have the same slope, the discharge may be expressed as

$$Q_1 = K_1\sqrt{S} \qquad Q_2 = K_2\sqrt{S}$$

or $$Q = (K_1 + K_2)\sqrt{S} \qquad \text{………} \qquad (1)$$

- in which the value of *K* is

$$K = \frac{1}{n} A R^{2/3}$$

- from Manning's formula and is a function of depth only for a given channel with fixed roughness. By computing K_1 and K_2 for different elevations of water surface, their sum may be taken and plotted against elevation.

- From this plot it is easy to determine the slope of energy grade line for a given depth and discharge from Eq. (1).

4 HYDRAULIC JUMP; STILLING BASINS

- **The relations among the variables V_1, y_1, V_2, y_2 for a hydraulic jump to occur in a horizontal rectangular channel are developed before. Another way of determining the conjugate depths for a given discharge is the $F + M$ method.**

- **The momentum equation applied to the free body of liquid between y_1 and y_2 (Fig. 12.4) is, for unit width ($V_1y_1 = V_2y_2 = q$),**

$$\frac{\gamma y_1^2}{2} - \frac{\gamma y_2^2}{2} = \rho q(V_2 - V_1) = \rho V_2^2 y_2 - \rho V_1^2 y_1$$

- **Rearranging gives**

$$\frac{\gamma y_1^2}{2} + \rho V_1^2 y_1 = \frac{\gamma y_2^2}{2} + \rho V_2^2 y_2 \qquad \textbf{(4.1)}$$

- **or**

$$F_1 + M_1 = F_2 + M_2 \qquad \textbf{(4.2)}$$

- **F is the hydrostatic force at the section and M is the momentum per second passing the section.**

- By writing *F* + *M* for a given discharge *q* per unit width

$$F + M = \frac{\gamma y^2}{2} + \frac{\rho q^2}{y} \qquad (4.3)$$

- a plot is made of *F* + *M* as abscissa against *y* as ordinate (Fig. 12.5) for q = 1 $m^3/s \cdot m$. Any vertical line intersecting the curve cuts it at two points having the same value of *F* + *M*; hence, they are conjugate depths.

- The value of *y* for minimum *F* + *M* is

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} \qquad (4.4)$$

- This depth is the *critical depth*, which is shown in the following section to be the depth of minimum energy. Therefore, the jump always occurs from rapid flow to tranquil flow.

Figure 4 Hydraulic jump in horizontal rectangular channel

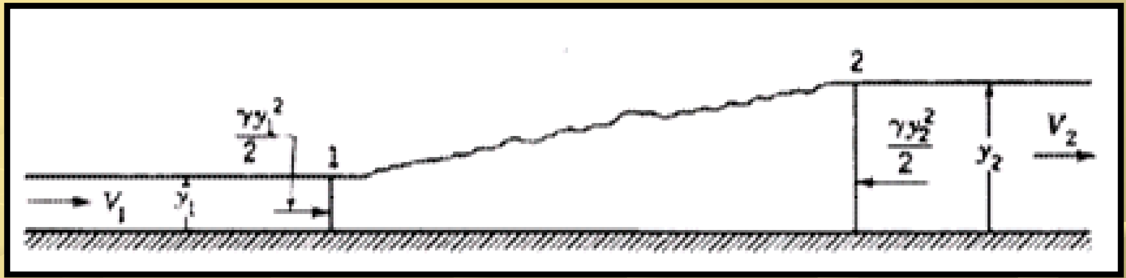
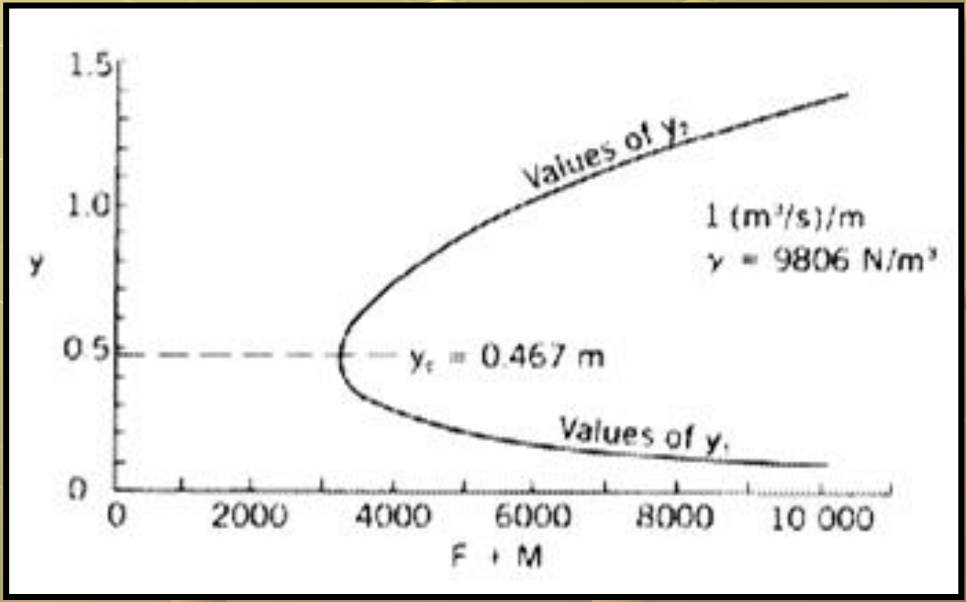


Figure 5 F + M curve for hydraulic jump



- **The conjugate depth are directly related to the Froude number before and after the jump,**

$$\boxed{\mathbf{F}_1 = \frac{V_1}{\sqrt{gy_1}}} \qquad \boxed{\mathbf{F}_2 = \frac{V_2}{\sqrt{gy_2}}} \qquad \textbf{(4.5)}$$

- **From the continuity equation**

$$\boxed{V_1^2 y_1^2 = gy_1^3 \mathbf{F}_1^2 = V_2^2 y_2^2 = gy_2^3 \mathbf{F}_2^2}$$

- **or**

$$\boxed{\mathbf{F}_1^2 y_1^3 = \mathbf{F}_2^2 y_2^3} \qquad \textbf{(4.6)}$$

- **From Eq. (12.4.1)**

$$\boxed{y_1^2\left(1 + 2\frac{V_1^2}{gy_1}\right) = y_2^2\left(1 + 2\frac{V_2^2}{gy_2}\right)}$$

- **Substituting from Eqs. (12.4.5) and (12.4.6) gives**

$$\boxed{(1 + 2\mathbf{F}_1^2)\mathbf{F}_1^{-4/3} = (1 + 2\mathbf{F}_2^2)\mathbf{F}_2^{-4/3}} \qquad \textbf{(4.7)}$$

- The value of F_2 in terms of F_1 is obtained from the hydraulic-jump equation

$$y_2 = \frac{-y_1}{2} + \sqrt{\left(\frac{y_1}{2}\right)^2 + 2\frac{V_1^2 y_1}{g}} \qquad \text{or} \qquad 2\frac{y_2}{y_1} = -1 + \sqrt{1 + 8\frac{V_1^2}{g y_1}}$$

- By Eqs. (4.5) and (4.6)

$$\mathbf{F}_2 = \frac{2\sqrt{2}\,\mathbf{F}_1}{(\sqrt{1 + 8\mathbf{F}_1^2} - 1)^{3/2}} \qquad (4.8)$$

- These equations apply only to a rectangular section.

- The Froude number is always greater than unity before the jump and less than unity after the jump.

Stilling Basins

- A stilling basin is a structure for dissipating available energy of flow below a spillway, outlet works, chute, or canal structure. In the majority of existing installations a hydraulic jump is housed within the stilling basin and used as the energy dissipator.

- This discussion is limited to rectangular basins with horizontal floors although sloping floors are used in some cases to save excavation. See table 12.1

- Baffle blocks are frequently used at the entrance to a basin to corrugate the flow. They are usually regularly spaced with gaps about equal to block widths.

- Sills, either triangular or dentated, are frequently employed at the downstream end of a basin to aid in holding the jump within the basin and to permit some shortening of the basin.

- The basin should be paved with high-quality concrete to prevent erosion and cavitation damage. No irregularities in floor or training walls should be permitted.

Table 1 Classification of the hydraulic jump as an effective energy dissipator

Froude number $\mathbf{F}_1(V_1/\sqrt{gy_1})$ entering the basin as follows:

At $\mathbf{F}_1 = 1$ to 1.7. Standing wave. There is only a slight difference in conjugate depths. Near $\mathbf{F}_1 = 1.7$ a series of small rollers develops.

At $\mathbf{F}_1 = 1.7$ to 2.5. Pre-jump. The water surface is quite smooth; the velocity is fairly uniform; and the head loss is low. No baffles required if proper length of pool is provided.

At $\mathbf{F}_1 = 2.5$ to 4.5. Transition. Oscillating action of entering jet, from bottom of basin to surface. Each oscillation produces a large wave of irregular period that can travel downstream for miles and damage earth banks and riprap. If possible, it is advantageous to avoid this range of Froude numbers in stilling basin design.

At $\mathbf{F}_1 = 4.5$ to 9. Range of good jumps. The jump is well balanced, and the action is at its best. Energy absorption (irreversibilities) ranges from 45 to 70 percent. Baffles and sills may be utilized to reduce length of basin.

At $\mathbf{F}_1 = 9$ upward. Effective but rough. Energy dissipation up to 85 percent. Other types of stilling basins may be more economical.

Example 2

- A hydraulic jump occurs downstream from a 15-m-wide sluice gate. The depth is 1.5 m, and the velocity is 20 m/s. Determine (a) the Froude number and the Froude number corresponding to the conjugate depth, (b) the depth and velocity after the jump, and (c) the power dissipated by the jump.

Solution

(a)

$$\mathbf{F}_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{20}{\sqrt{9.806 \times 1.5}} = 5.215$$

- From Eq. (12.4.8)

$$\mathbf{F}_2 = \frac{2\sqrt{2} \times 5.215}{(\sqrt{1 + 8 \times 5.215^2} - 1)^{3/2}} = 0.2882$$

(b)

$$\mathbf{F}_2 = \frac{V_2}{\sqrt{gy_2}} \qquad V_2 y_2 = V_1 y_1 = 1.5 \times 20 = 30 \text{ m}^2/\text{s}$$

- **Then**

$$V_2^2 = \mathbf{F}_2^2 g y_2 = \mathbf{F}_2^2 g \frac{30}{V_2}$$

- **and**

$$V_2 = (0.2882^2 \times 9.806 \times 30)^{1/3} = 2.90 \text{ m/s}$$

$$y_2 = 10.34 \text{ m}$$

(c)

- **Form Eq. (3.11.24), the head loss *h_j* in the jump is**

$$h_j = \frac{(y_2 - y_1)^3}{4y_1 y_2} = \frac{(10.34 - 1.50)^3}{4 \times 1.5 \times 10.34} = 11.13 \text{ m} \cdot \text{N/N}$$

- **The power dissipated is**

$$\text{power} = \gamma Q h_j = 9806 \times 15 \times 30 \times 11.13 = 49.1 \text{ MW}$$

5. SPECIFIC ENERGY; CRITICAL DEPTH

- The energy per unit weight E_s with elevation datum taken as the bottom of the channel is called the *specific energy*. It is plotted vertically above the channel floor,

$$\boxed{E_s = y + \frac{V^2}{2g}} \qquad (5.1)$$

- A plot of specific energy for a particular case is shown in Fig. 12.6. In a rectangular channel, in which q is the discharge per unit width, with Vy = q,

$$\boxed{E_s = y + \frac{q^2}{2gy^2}} \qquad (5.2)$$

- It is of interest to note how the specific energy varies with the depth for a constant discharge (Fig. 7).

- For small values of y the curve goes to infinity along the E_s axis, while for large values of y the velocity head term is negligible and the curve approaches the 45^0 line $E_s = y$ asymptotically.

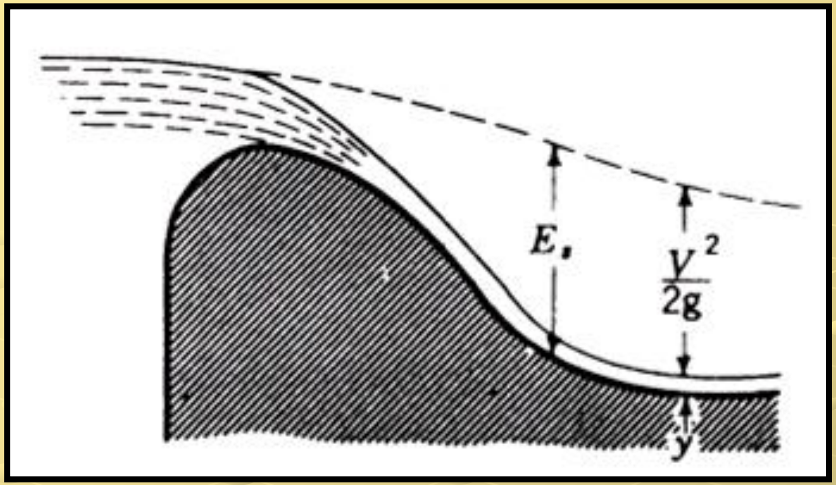
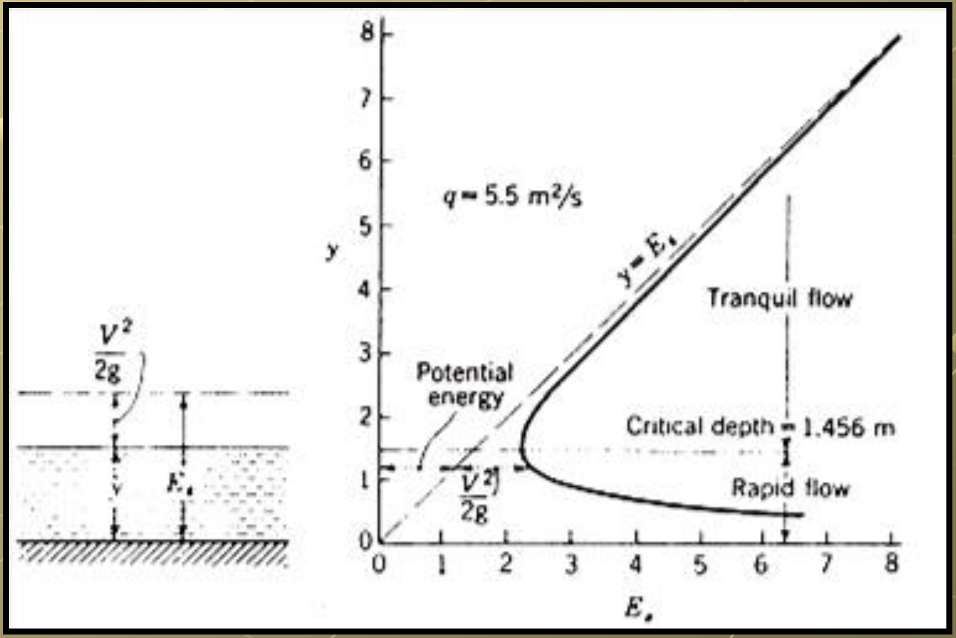


Figure 6 Example of specific energy

Figure 7 Specific energy required for flow of a given discharge at various depths.



- The value of y for minimum E_s is obtained by setting dE_s/dy equal to zero, from Eq. (5.2), holding q constant,

$$\frac{dE_s}{dy} = 0 = 1 - \frac{q^2}{gy^3}$$

- or

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} \qquad (5.3)$$

- The depth for minimum energy y_c is called *critical depth*. Eliminating q^2 in Eqs. (5.2) and (5.3) gives

$$E_{s_{min}} = \frac{3}{2}y_c \qquad (5.4)$$

- showing that the critical depth is two-thirds or the specific energy. Eliminating E_s in Eqs. (5.1) and (5.4) gives

$$V_c = \sqrt{gy_c} \qquad (5.5)$$

- Another method of arriving at the critical condition is to determine the maximum discharge q that could occur for a given specific energy. The resulting equations are the same as Eqs. (5.3) to (5.5).

- For nonrectangular cross sections, as illustrated in Fig. 8, the specific-energy equation takes the form

$$\boxed{E_s = y + \frac{Q^2}{2gA^2}} \tag{5.6}$$

- in which *A* is the cross-sectional area. To find the critical depth

$$\boxed{\frac{dE_s}{dy} = 0 = 1 - \frac{Q^2}{gA^3}\frac{dA}{dy}}$$

- From Fig. 8, the relation between *dA* and *dy* is expressed by

$$\boxed{dA = T\,dy}$$

- in which *T* is the width of the cross section at the liquid surface. With this relation

$$\boxed{\frac{Q^2}{gA_c^3}T_c = 1} \tag{5.7}$$

- **The critical depth must satisfy this equation. Eliminating *Q* in Eqs. (5.6) and (5.7) gives**

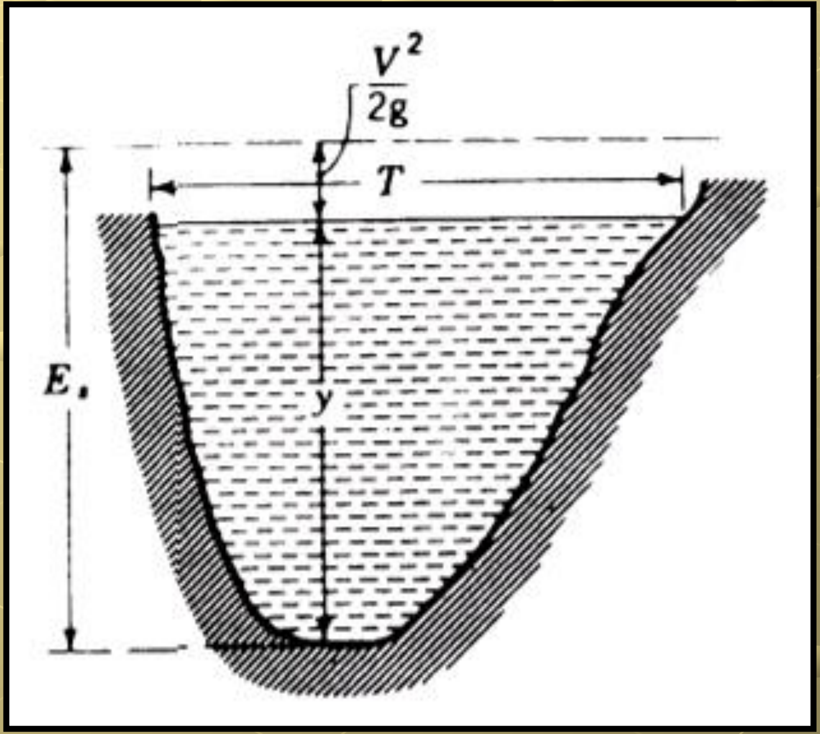
$$\boxed{E_s = y_c + \frac{A_c}{2T_c}} \qquad \textbf{(5.8)}$$

- **This equation shows that the minimum energy occurs when the velocity head is one-half the average depth *A/T*. Equation (5.7) may be solved by trail for irregular sections by plotting**

$$\boxed{f(y) = \frac{Q^2 T}{g A^3}}$$

- **Critical depth occurs for that value of *y* which makes *f*(*y*) = 1.**

Figure 8 **Specific energy for a nonrectangular section**



Example 3

- **Determine the critical depth for 10 m^3/s flowing in a trapezoidal channel with bottom width 3 m and side slopes 1 horizontal to 2 vertical (1 on 2).**

Solution

$$A = 3y + \frac{y^2}{2} \qquad T = 3 + y$$

- **Hence**

$$f(y) = \frac{10^2(3+y)}{9.806(3y + y^2/2)^3} = \frac{10.198(3+y)}{(3y+0.5y^2)^3} = 1.0$$

- **By trail**

y	2.0	1.2	0.8	1.0	0.99	0.98	0.985	0.984
$f(y)$	0.1	0.53	1.92	0.95	0.982	1.014	0.998	1.0014

- **The critical depth is 0.984 m. This trail solution is easily carried out by programmable calculator.**

6. TRANSITIONS

- **At entrances to channels and at changes in cross section and bottom slope, the structure that conducts the liquid from the upstream section to the new section is a *transition*.**

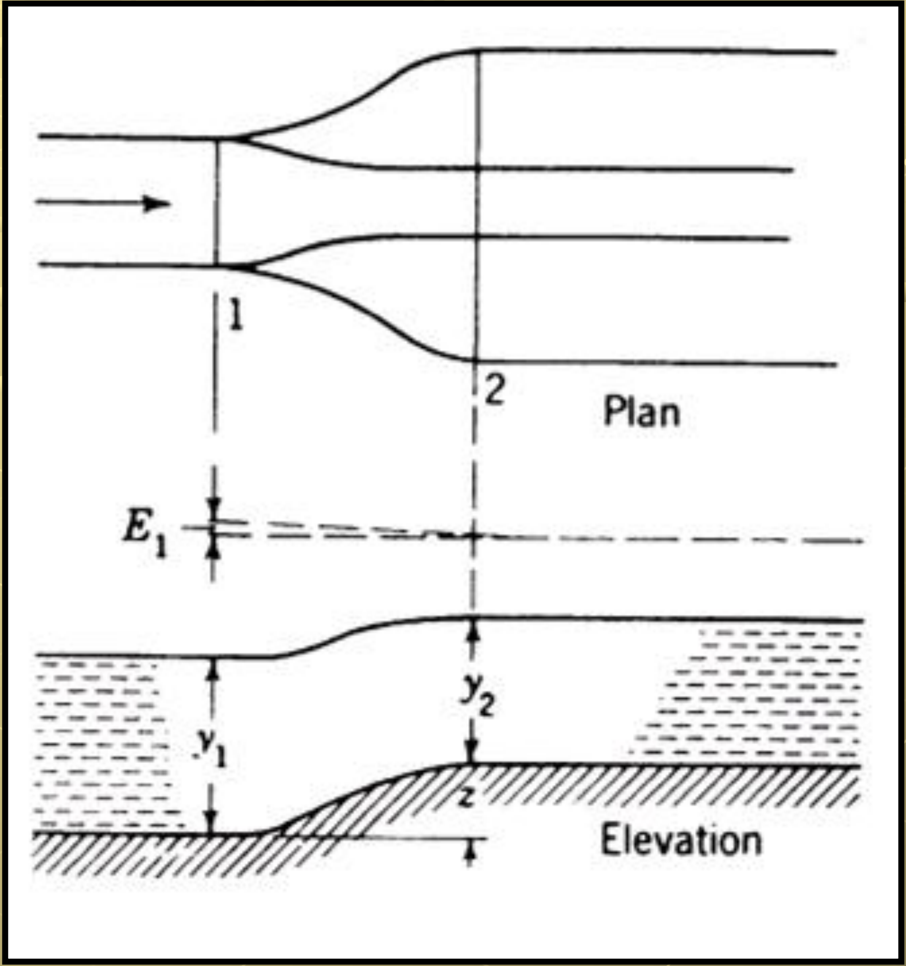
- **Its purpose is to change the shape of flow and surface profile in such a manner that minimum losses result.**

- **A transition for tranquil flow from a rectangular channel to a trapezoidal channel is illustrated in Fig. 10. Applying the energy equation from section 1 to section 2 gives**

$$\boxed{\frac{V_1^2}{2g} + y_1 = \frac{V_2^2}{2g} + y_2 + z + E_1} \qquad \textbf{(6.1)}$$

- **In general, the sections and depths are determined by other considerations, and *z* must be determined for the expected available energy loss E_1 .**

Figure 10 **Transition from rectangular channel to trapezoidal channel for tranquil flow**



Example 5

- **In Fig. 12.10, 11 m^3/s flows through the transition; the rectangular section 2.4 m wide; and y_1 = 2.4 m. The trapezoidal section is 1.8 m wide at the bottom with side slopes 1 : 1, and y_2 = 2.25 m. Determine the rise *z* in the bottom through the transition.**

- ***Solution***

$$V_1 = \frac{11}{2.4 \times 2.4} = 1.91 \qquad \frac{V_1^2}{2g} = 0.186 \qquad A_2 = 9.11 \text{ m}^2$$

$$V_2 = \frac{11}{9.11} = 1.21 \qquad \frac{V_2^2}{2g} = 0.0743 \qquad E_1 = 0.3\,(0.186 - 0.0743) = 0.0335$$

- **Substituting into Eq. (6.1) gives**

$$z = 0.186 + 2.4 - 0.0743 - 2.25 - 0.0335 = 0.228 \text{ m}$$

- **The critical-depth meter is an excellent device for measuring discharge in an open channel. The relations for determination of discharge are worked out for a rectangular channel of constant width, Fig. 11, with a raised floor over a reach of channel about 3y_c long.**

- **Applying the energy equation from section 1 to the critical section (exact location unimportant), including the transition-loss term, gives**

$$\boxed{\frac{V_1^2}{2g} + y_1 = z + y_c + \frac{V_c^2}{2g} + \frac{1}{10}\left(\frac{V_c^2}{2g} - \frac{V_1^2}{2g}\right)}$$

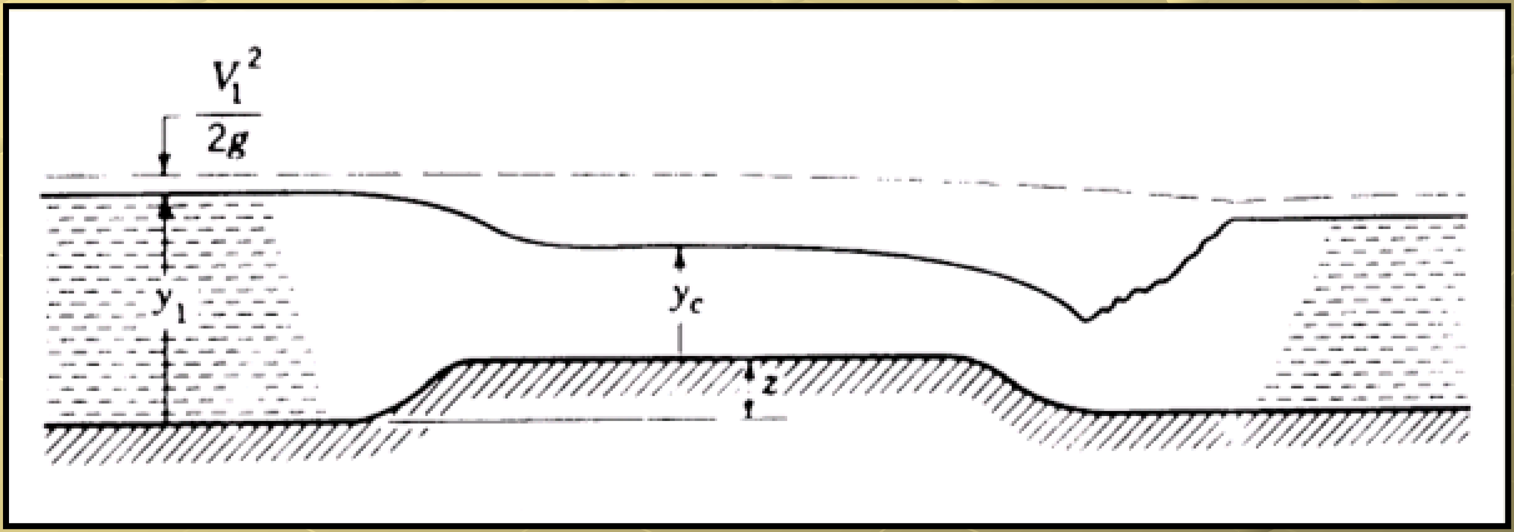
- **Since**

$$\boxed{y_c + \frac{V_c^2}{2g} = E_c \qquad \frac{V_c^2}{2g} = \frac{E_c}{3}}$$

- **In which E_c is the specific energy at critical depth,**

$$\boxed{y_1 + 1.1\frac{V_1^2}{2g} = z + 1.033E_c} \qquad \textbf{(6.2)}$$

Figure 11 Critical-depth meter



- **From Eq. (5.3)**

$$y_c = \frac{2}{3} E_c = \left(\frac{q^2}{g}\right)^{1/3} \qquad (6.3)$$

- **In Eqs. (6.2) and (6.3) *E_c* is eliminated and the resulting equation solved for q,**

$$q = 0.517 g^{1/2} \left(y_1 - z + 1.1 \frac{V_1^2}{2g}\right)^{3/2}$$

- **Since q = *$V_1 y_1$*, *V_1* can be eliminated,**

$$q = 0.517 g^{1/2} \left(y_1 - z + \frac{0.55}{g} \frac{q^2}{y_1^2}\right)^{3/2} \qquad (6.4)$$

- **The equation is solved by trial. As *y_1* and *z* are known and the right-hand term containing *q* is small, it may first be neglected for an approximate *q*. A value a little larger than the approximate *q* may be substituted on the right-hand side. When the two *q* 's are the same the equation is solved.**
- **Experiments indicate that accuracy within 2 to 3 percent may be expected.**

Example 6

- In a critical-depth meter 2 m wide with *z* = 0.3 m the depth y_1 is measured to be 0.75 m. Find the discharge.

Solution

$$q = 0.517(9.806^{1/2})(0.45^{3/2}) = 0.489\ m^2/s$$

- As a second approximation let *q* be 0.50,

$$q = 0.517(9.806^{1/2})\left(0.45 + \frac{0.55}{9.806} \times 0.5^2\right)^{3/2} = 0.512\ m^2/s$$

- And as a third approximation, 0.513,

$$q = 0.517(9.806^{1/2})\left(0.45 + \frac{0.55}{9.806} \times 0.513^2\right)^{3/2} = 0.513\ m^2/s$$

- Then

$$Q = 2 \times 0.513 = 1.026\ m^3/s$$

7 .GRADUALLY VARIED FLOW

- Gradually varied flow is steady nonuniform flow of a special class. The depth, area, roughness, bottom slope, and hydraulic radius change very slowly (if at all) along the channel.

- Solving Eq. (12.2.1) for the head loss per unit length of channel produces

$$S = -\frac{\Delta E}{\Delta L} = \left(\frac{nQ}{C_m A R^{2/3}}\right)^2 \qquad (7.1)$$

- in which S is now the slope of the energy grade line or, more specifically, the sine of the angle the energy grade line makes with the horizontal.

- Computations of gradually varied flow may be carried out either by the *standard-step method* or by *numerical integration*. Horizontal channels of great width are treated as a special case that may be integrated.

Standard-Step Method

- Applying the energy equation between two sections a finite distance ΔL apart (Fig. 12.12), including the loss term, gives

$$\frac{V_1^2}{2g} + S_0\,\Delta L + y_1 = \frac{V_2^2}{2g} + y_2 + S\,\Delta L \qquad (7.2)$$

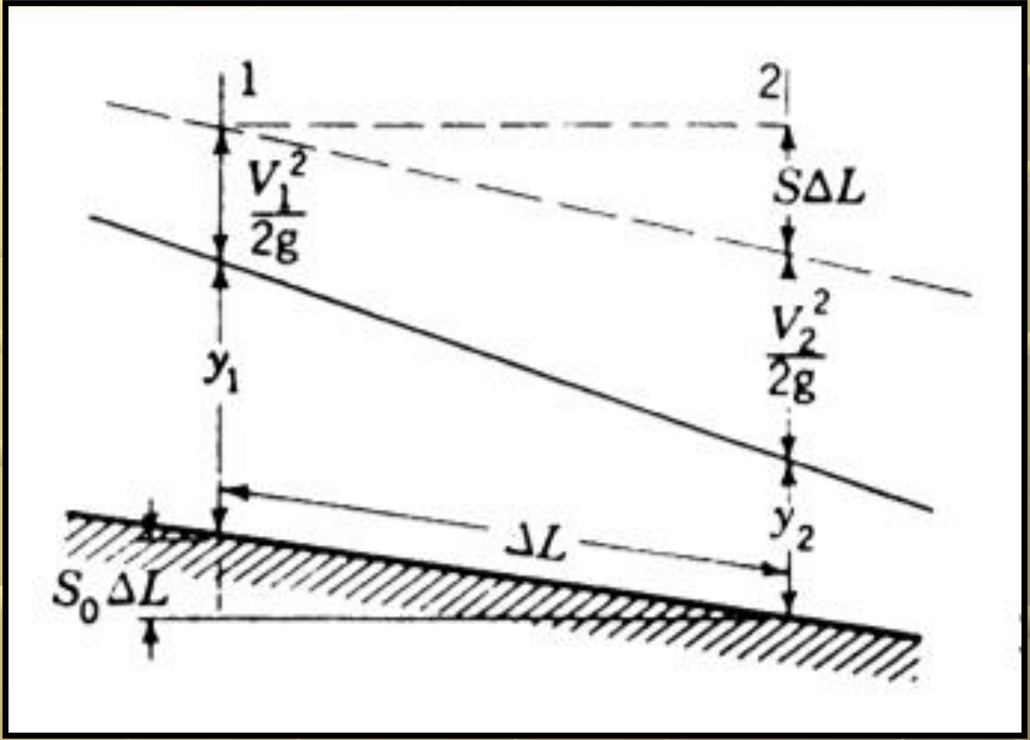
- Solving for the length of reach gives

$$\Delta L = \frac{(V_1^2 - V_2^2)/2g + y_1 - y_2}{S - S_0} \qquad (7.3)$$

- If conditions are known at one section, e.g., section 1, and the depth y_2 is wanted a distance ΔL away, a trial solution is required. The procedure is as follows:

1. Assume a depth y_2; then compute A_2, V_2.
2. For the assumed y_2 find an average y, P, and A for the reach and compute S.
3. Substitute in Eq. (12.7.3) to compute ΔL.
4. If ΔL is not correct, assume a new y_2 and repeat the procedure.

Figure 12 Gradually varied flow



- The standard-step method is easily followed with the programmable calculator if about 20 memory storage spaces and about 100 program steps are available.

- In the first trial y_2 is used to evaluate ΔL_{new}. Then a linear proportion yields a new trial y_{2new} for the next step; thus

$$\boxed{\frac{y_1 - y_2}{\Delta L_{new}} = \frac{y_1 - y_{2new}}{\Delta L_{given}}}$$

or

$$\boxed{y_{2new} = y_1 + (y_2 - y_1)\frac{\Delta L_{given}}{\Delta L_{new}}}$$

- A few iterations yield complete information on section 2.

Example 7

- At section 1 of a canal the cross section is trapezoidal, b_1 = 10 m, m_1 = 2, y_1 = 7 m, and at section 2, downstream 200 m, the bottom is 0.08 m higher than at section 1, b_2 = 15 m, and m_2 = 3. Q = 200 m^3/s, n = 0.035. Determine the depth of water at section 2.

Solution

$$A_1 = b_1 y_1 + m_1 y_1^2 = 10 \times 7 + 2 \times 7^2 = 168 \text{ m}^2 \qquad V_1 = \tfrac{200}{168} = 1.19 \text{ m/s}$$

$$P_1 = b_1 + 2y_1\sqrt{m_1^2 + 1} = 10 + 2 \times 7\sqrt{2^2 + 1} = 41.3 \text{ m}$$

$$S_0 = -\frac{0.08}{200} = -0.0004$$

- Since the bottom has an adverse slope, i.e.. it is rising in the downstream direction, and since section 2 is larger than section 1, y_2 is probably less than y_1. Assume y_2 = 6.9 m; then

$$A_2 = 15 \times 6.9 + 3 \times 6.9^2 = 246 \text{ m}^2 \qquad V_2 = \tfrac{200}{246} = 0.813 \text{ m/s}$$

- and

$$P_2 = 15 + 2 \times 6.9\sqrt{10} = 58.6 \text{ m}$$

- **The average A = 207 and average wetted perimeter P = 50.0 are used to find an average hydraulic radius for the reach, R = 4.14 m. Then**

$$S = \left(\frac{nQ}{C_m A R^{2/3}}\right)^2 = \left(\frac{0.035 \times 200}{1.0 \times 207 \times 4.14^{2/3}}\right)^2 = 0.000172$$

- **Substituting into Eq. (7.3) gives**

$$\Delta L = \frac{(1.19^2 - 0.813^2)/(2 \times 9.806) + 7 - 6.9}{0.000172 + 0.0004} = 242 \text{ m}$$

- **A larger y_2, for example, 6.92 m, would bring the computed value of length closer to the actual length.**

Numerical Integration Method

- A more satisfactory procedure, particularly for flow through channels having a constant shape of cross section and constant bottom slope, is to obtain a differential equation in terms of *y* and *L* and then perform the integration numerically.

- When ΔL is considered as an infinitesimal in Fig. 12.12, the rate of change of available energy equals rate of head loss $-\Delta E/\Delta L$ given by Eq. (12.7.1), or

$$\frac{d}{dL}\left(\frac{V^2}{2g} + z_0 - S_0 L + y\right) = -\left(\frac{nQ}{C_m A R^{2/3}}\right)^2 \tag{12.7.4}$$

- in which $z_0 - S_0 L$ is the elevation of bottom of channel at L, z_0 is the elevation of bottom at $L = 0$, and L is measured positive in the downstream direction.

- After performing the differentiation,

$$-\frac{V}{g}\frac{dV}{dL} + S_0 - \frac{dy}{dL} = \left(\frac{nQ}{C_m A R^{2/3}}\right)^2 \tag{12.7.5}$$

- Using the continuity equation $VA = Q$ leads to

$$\boxed{\frac{dV}{dL} A + V \frac{dA}{dL} = 0}$$

- And expression $dA = T\,dy$, in which T is the liquid–surface width of the cross section, gives

$$\boxed{\frac{dV}{dL} = -\frac{VT}{A}\frac{dy}{dL} = -\frac{QT}{A^2}\frac{dy}{dL}}$$

- Substituting for V in Eq. (12.7.5) yields

$$\boxed{\frac{Q^2}{gA^3} T \frac{dy}{dL} + S_0 - \frac{dy}{dL} = \left(\frac{nQ}{C_m A R^{2/3}}\right)^2}$$

- And solving for dL gives

$$\boxed{dL = \frac{1 - Q^2 T/gA^3}{S_0 - (nQ/C_m A R^{2/3})^2}\, dy} \qquad \textbf{(7.6)}$$

- **After integrating,**

$$L = \int_{y_1}^{y_2} \frac{1 - Q^2T/gA^3}{S_0 - (nQ/C_m AR^{2/3})^2}\, dy \qquad \textbf{(7.7)}$$

- **In which *L* is the distance between the two sections having depths y_1 and y_2.**

- **When the numerator of the integrand is zero, critical flow prevails; there is no change in *L* for a change in *y* (neglecting curvature of the flow and nonhydrostatic pressure distribution at this section). Since this is not a case of gradual change in depth, the equations are not accurate near critical depth.**

- **When the denominator of the integrand is zero, uniform flow prevails and there is no change in depth along the channel. The flow is at *normal depth*.**

- For a channel of prismatic cross section, constant *n* and S_0, the integrand becomes a function of *y* only,

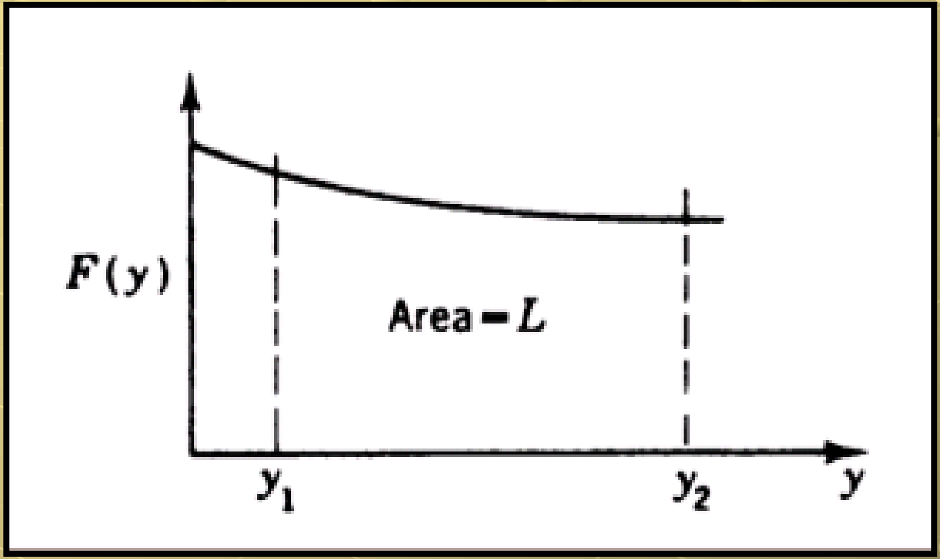
$$F(y) = \frac{1 - Q^2T/gA^3}{S_0 - (nQ/C_m AR^{2/3})^2}$$

- and the equation can be integrated numerically by plotting *F*(*y*) as ordinate against *y* as abscissa.

- The area under the curve (Fig 13) between two values of *y* is the length *L* between the sections, since

$$L = \int_{y_1}^{y_2} F(y)\, dy$$

Figure 13 **Numerical integration of equation for gradually varied flow**



Example 8

- A trapezoidal channel, b = 3 m, m = 1, n = 0.014, S_0 = 0.001, carries 28 m^3/s. If the depth is 3 m at section 1, determine the water-surface profile for the next 700 m downstream.

Solution

- To determine whether the depth increases or decreases, the slope of the energy grade line at section 1 is computed using Eq. (12.7.1)

$$A = by + my^2 = 3 \times 3 + 1 \times 3^2 = 18 \text{ m}^2$$

$$P = b + 2y\sqrt{m^2 + 1} = 11.485 \text{ m}$$

- and

$$R = \frac{18}{11.485} = 1.567 \text{ m}$$

- Then

$$S = \left(\frac{0.014 \times 28}{18 \times 1.567^{2/3}}\right)^2 = 0.00026$$

- **Substituting into Eq. (5.7) the values *A*, *Q* and *T* = 9 m gives Q^2T/gA^3 = 0.12, showing that the depth is above critical. With the depth greater than critical and the energy grade line less steep than the bottom of the channel, the specific energy is increasing.**

- **When the specific energy increases above critical, the depth of flow increases. Δ*y* is then positive. Substituting into Eq. (7.6) yields**

$$L = \int_3^y \frac{1 - 79.95T/A^3}{0.001 - 0.1537/(A^2R^{4/3})}\,dy$$

- **The following table evaluates the terms of the integrand.**

y	A	P	R	T	Numerator	$10^6 \times$ Denominator	$F(y)$	L
3	18	11.48	1.57	9	0.8766	739	1185	0
3.2	19.84	12.05	1.65	9.4	0.9038	799	1131	231.6
3.4	21.76	12.62	1.72	9.8	0.9240	843	1096	454.3
3.6	23.76	13.18	1.80	10.2	0.9392	876	1072	671.1
3.8	25.84	13.75	1.88	10.6	0.9509	901	1056	883.9

- **The integral $\int F(y)\, dv$ can be evaluated by plotting the curve and taking the area under it between $y = 3$ and the following values of y. As $F(y)$ does not vary greatly in this example, the average of $F(y)$ can be used for each reach (the trapezoidal rule); and when it is multiplied by Δy, the length of reach is obtained. Between $y = 3$ and $y = 3.2$**

$$\boxed{\frac{1185 + 1131}{2} 0.2 = 231.6}$$

- **Between $y = 3.2$ and $y = 3.4$**

$$\boxed{\frac{1131 + 1096}{2} 0.2 = 222.7}$$

- **and so on. Five points on it are known, so the water surface can be plotted. A more accurate way of summing $F(y)$ to obtain L is by use of Simpson's rule.**
- **The procedure used is equivalent to a Runge-Kutta second-order solution of a differential equation.**

- **A programmable calculator (about 20 memory storage spaces and 75 program steps) was used to carry out this solution. By taking $\Delta y = 0.1$ m in place of 0.2 m, the length to $y = 3.6$ m is 0.6 m less.**

Horizontal Channels of Great Width

- For channels of great width the hydraulic radius equals the depth; and for horizontal channel floors, S_0 = 0; hence, Eq. (7.7) can be simplified.

- The width may be considered as unity; that is, T = 1, Q = q and A = y, R = y; thus

$$L = -\int_{y_1}^{y} \frac{1 - q^2/gy^3}{n^2q^2/C_m^2 y^{10/3}}\, dy \tag{7.8}$$

- or, after performing the integration,

$$L = -\frac{3}{13}\left(\frac{C_m}{nq}\right)^2 (y^{13/3} - y_1^{13/3}) + \frac{3}{4g}\left(\frac{C_m}{n}\right)^2 (y^{4/3} - y_1^{4/3}) \tag{7.9}$$

- The computation of water-surface profiles with the aid of a digital computer is discussed after the various types of gradually varied flow profiles are classified.

Example 9

- **After contracting below a sluice gate water flows onto a wide horizontal floor with a velocity of 15 m/s and a depth of 0.7 m. Find the equation for the water-surface profile, *n* = 0.015.**

Solution

- **From Eq. (7.9), with *x* replacing *L* as distance from section 1, where y_1 = 0.7, and with $q = 0.7 \times 15 = 10.5\ m^2/s$.**

$$x = -\frac{3}{13}\left(\frac{1}{0.015 \times 10.5}\right)^2 (y^{13/3} - 0.7^{13/3}) + \frac{3}{4 \times 9.806}\left(\frac{1}{0.015}\right)^2 (y^{4/3} - 0.7^{4/3})$$
$$= -209.3 - 9.30y^{13/3} + 340y^{4/3}$$

- **Critical depth occurs [Eq. (12.5.3)] at**

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{10.5^2}{9.806}\right)^{1/3} = 2.24\ \text{m}$$

- **The depth must increase downstream, since the specific energy decreases, and the depth must move toward the critical value for less specific energy. The equation does not hold near the critical depth because of vertical accelerations that have been neglected in the derivation of gradually varied flow.**

- **If the channel is long enough for critical depth to be attained before the end of the channel, the high-velocity flow downstream from the gate may be drowned or a jump may occur.**

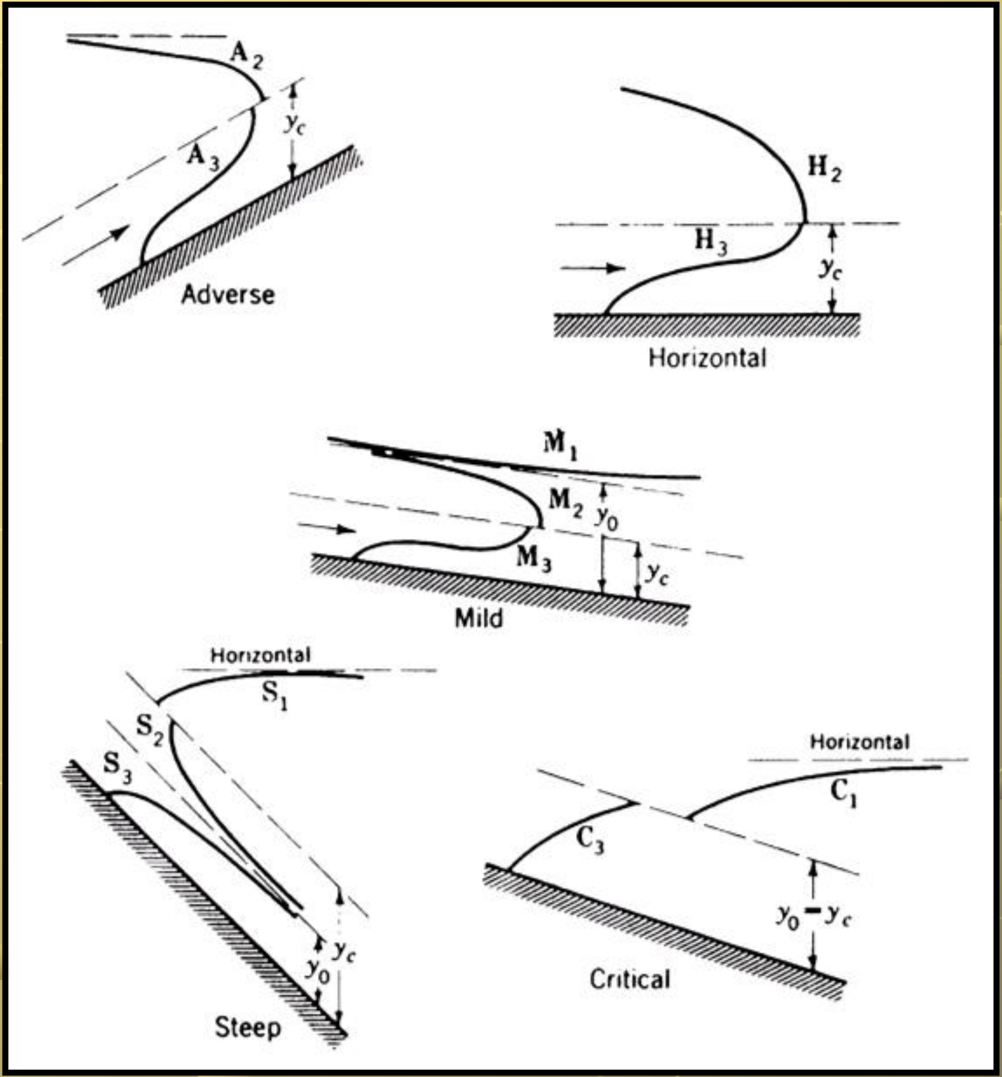
- **The water-surface calculation for the subcritical flow must begin with critical depth at the downstream end of the channel.**

8. CLASSIFICATION OF SURFACE PROFILES

- **A study of Eq. (12.7.7) reveals many types of surface profiles, each with definite characteristics. The bottom slope is classified as *adverse*, *horizontal*, *mild*, *critical*, and *steep*; and, in general, the flow can be above the normal depth or below the normal depth, and it can be above critical depth or below critical depth.**

- **The various profiles are plotted in Fig. 12.14; the procedures used are discussed for the various classifications in the following paragraphs. A very wide channel is assumed in the reduced equations which follow, with $R = y$.**

Figure 14 Typical liquid-surface profiles



Adverse Slope Profiles

- When the channel bottom rises in the direction of flow (S_0 is negative), the resulting surface profiles are said to be adverse. There is no normal depth, but the flow may be either below or above critical depth.

- Below critical depth the numerator is negative, and Eq. (7.6) has the form

$$\boxed{dL = \frac{1 - C_1/y^3}{S_0 - C_2/y^{10/3}}\,dy}$$

- where C_1 and C_2, are positive constants.

- Here $F(y)$ is positive and the depth increases downstream. This curve is labeled A_3 and shown in Fig. 12.14. For depths greater than critical depth, the numerator is positive, and $F(y)$ is negative; i.e., the depth decreases in the downstream direction. For y very large, $dL/dy = 1/S_0$, which is a horizontal asymptote for the curve.

- At $y = y_c$, dL/dy is 0, and the curve is perpendicular to the critical-depth line. This curve is labeled A_2.

Horizontal Slope Profiles

- For a horizontal channel $S_0 = 0$, the normal depth is infinite and flow may be either below critical depth or above critical depth.

- The equation has the form

$$\boxed{dL = -Cy^{1/3}(y^3 - C_1)\,dy}$$

- For y less than critical, dL/dy is positive, and the depth increases downstream. It is labeled H_3.

- For y greater than critical (H_2 curve), dL/dy is negative, and the depth decreases downstream. These equations are integrable analytically for very wide channels.

Mild Slope Profiles

- A mild slope is one on which the normal flow is tranquil i.e., where normal depth y_0 is greater than critical depth. Three profiles may occur, M_l, M_2, M_3, for depth above normal, below normal and above critical and below critical, respectively.

- For the M_1 curve, dL/dy is positive and approaches $1/S_0$ for very large y; hence, the M_1 curve has a horizontal asymptote downstream. As the denominator approaches zero as y approaches y_0 , the normal depth is an asymptote at the upstream end of the curve.

- Thus, dL/dy is negative for the M_2 curve, with the upstream asymptote the normal depth, and $dL/dy = 0$ at critical. The M_3 curve has an increasing depth downstream, as shown.

Critical Slope Profiles

- **When the normal depth and the critical depth are equal, the resulting profiles labeled C_1 and C_3 for depth above and below critical, respectively.**

- **The equation has the form**

$$\boxed{dL = \frac{1}{S_0}\frac{1 - b/y^3}{1 - b_1/y^{10/3}}\,dy}$$

- **with both numerator and denominator positive for C_1 and negative for C_3. Therefore, the depth increases downstream for both.**

- **For large y, *dL/dy* approaches $1/S_0$; hence, a horizontal line is an asymptote. The value of *dL/dy* at critical depth is $0.9/S_0$; hence, curve C_1 is convex upward. Curve C_3 also is convex upward, as shown.**

Steep Slope Profiles

- When the normal flow is rapid in a channel (normal depth less than critical depth), the resulting profiles S_1, S_2, S_3 are referred to as steep profiles:
  - S_1 is above the normal and critical,
  - S_2 between critical and normal, and
  - S_3 below normal depth.

- For curve S_1 both numerator and denominator are positive, and the depth increases downstream approaching a horizontal asymptote. For curve S_2 the numerator is negative and the denominator positive but approaching zero at $y = y_0$. The curve approaches the normal depth asymptotically. The S_3 curve has a positive *dL/dy* as both numerator and denominator are negative. It plots as shown on Fig. 14.

- It should be noted that a given channel may be classified as mild for one discharge, critical for another discharge, and steep for a third discharge, since normal depth and critical depth depend upon different functions of the discharge.

12.9 CONTROL SECTIONS

- A small change in downstream conditions cannot be relayed upstream when the depth is critical or less than critical; hence, downstream conditions do not control the flow. All rapid flows are controlled by upstream conditions, and computations of surface profiles must be started at the upstream end of a channel.

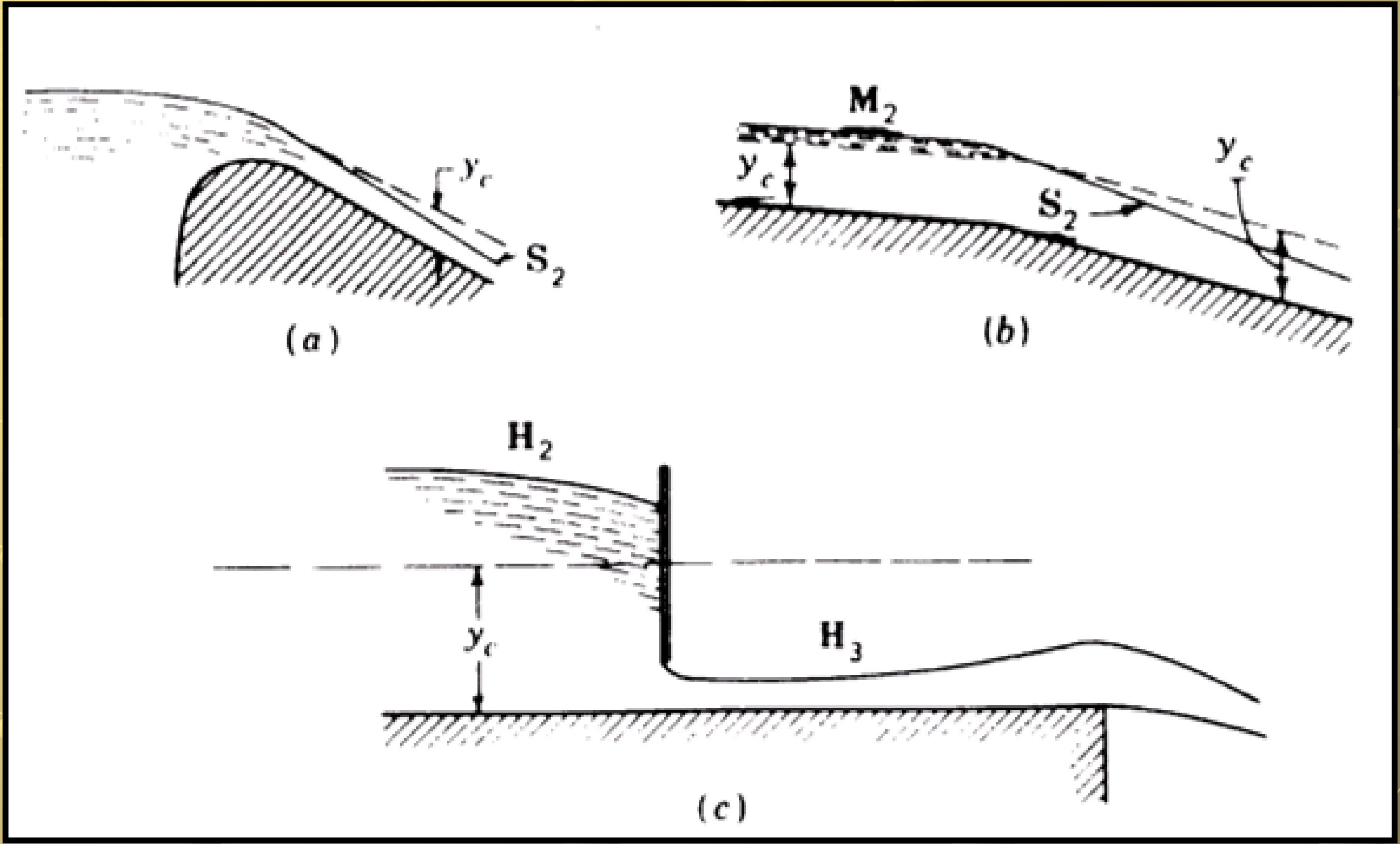
- Tranquil flows are affected by small changes in downstream conditions and therefore are controlled by them. Tranquil-flow computations must start at the downstream end of a reach and be carried upstream.

- Control sections occur at entrances and exits to channels and at changes in channel slopes, under certain conditions. In Fig. 15*a* the flow passes through critical at the entrance to a channel, and depth can be computed there for a given discharge.

- In Fig. 15*b* a change in channel slope from mild to steep causes the flow to pass through critical at the break in grade. Computations proceed both upstream and downstream from the control section at the break in grade.

- In Fig. 15*c* a gate in a horizontal channel provides control both upstream and downstream from it.

Figure 15 Channel control sections



- **The hydraulic jump occurs whenever the conditions required by the momentum equation are satisfied. In Fig. 16, liquid issues from under a gate in rapid flow along a horizontal channel. If the channel were short enough, the flow could discharge over the end of the channel as an H_3 curve.**

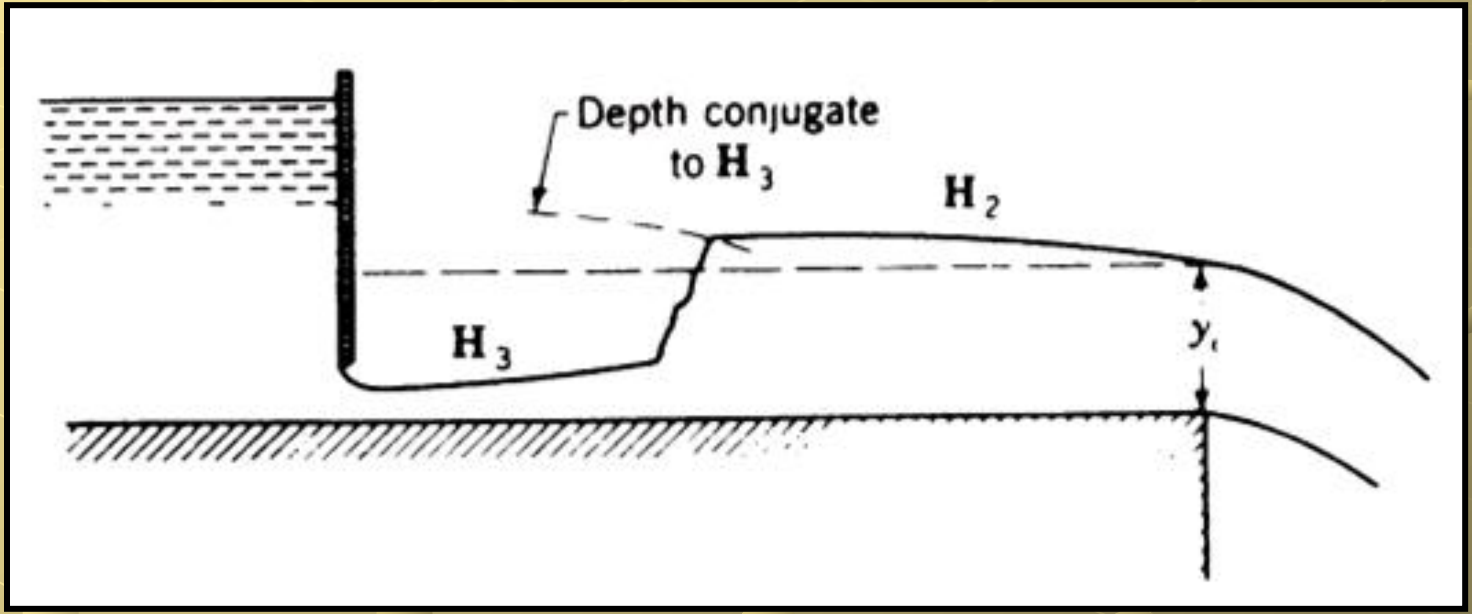
- **With a longer channel, however, the jump occurs, and the resulting profile consists of pieces of H_3 and H_2 curves with the jump in between. In computing these profiles for a known discharge, the H_3 curve is computed, starting at the gate (contraction coefficient must be known) and proceeding downstream until it is clear that the depth will reach critical before the end of the channel is reached.**

- **Then the H_2 curve is computed, starting with critical depth at the end of the channel and proceeding upstream. The depths conjugate to those along H_3 are computed and plotted as shown.**

- **The channel may be so long that the H_2 curve is everywhere greater than the depth conjugate to H_3. A *drowned jump* then occurs, with H_2 extending to the gate.**

- **All sketches are drawn to a greatly exaggerated vertical scale, since usual channels have small bottom slopes.**

Figure 16 Hydraulic jump between two control sections



10 COMPUTER CALCULATION OF GRADUALLY VARIED FLOW

- The program, listed in Fig. 17, calculates the steady gradually varied water-surface profile in any prismatic rectangular, symmetric trapezoidal or triangular channel.

- Input data include the specification of the system of units (SI or USC) in the first columns of the data, followed by the channel dimensions, discharge, and water-surface control depth on the second card.

- If the control depth is left blank or set to zero in data, it is automatically assumed to be the critical depth in the program.

- For subcritical flow the control is downstream, and distances are measured in the upstream direction. For supercritical flow the control depth is upstream, and distances are measured in the downstream direction.

Figure 17. **FORTRAN program for water-surface profiles**

```
C WATER SURFACE PROFILE IN RECT, TRAPEZOIDAL OR TRIANGULAR CHANNEL.
C XL=LENGTH, B=BOT WIDTH, Z=SIDE SLOPE,RN=MANNING N,SO=BOT SLOPE,Q=FLOW.
C YCONT=CONTROL DEPTH. IF YCONT=0. IN DATA, YCONT IS SET EQUAL TO YC.
C IN SUBCRITICAL FLOW, CONTROL IS DOWNSTREAM AND DISTANCES ARE MEASURED
C IN THE UPSTREAM DIRECTION
C IN SUPERCRITICAL FLOW, CONTROL IS U.S. AND DISTANCES ARE MEASURED D.S.
      DATA ISI/'SI'/
      AREA(YY)=YY*(B+Z*YY)
      PER(YY)=B+2.*YY*SQRT(1.+Z*Z)
      YCRIT(YY)=1.-Q*Q*(B+2.*Z*YY)/(G*AREA(YY)**3)
      YNORM(YY)=1.-Q*Q*CON/(AREA(YY)**3.333/PER(YY)**1.333)
      DL(YY)=YCRIT(YY)/(YNORM(YY)*SO)
      FPM(YY)=GAM*(YY*YY*(B*.5+Z*YY/3.)+Q*Q/(G*AREA(YY)))
      ENERGY(YY)=YY+Q*Q/(2.*G*AREA(YY)**2)
 5    READ (5,7,END=99) IUNIT,XL,B,Z,RN,SO,Q,YCONT
 7    FORMAT (A2/F10.1,3F10.4,F10.6,2F10.3)
      IF (IUNIT.EQ.ISI) GO TO 10
      GAM=62.4
      G=32.2
      CON=(RN/1.486)**2/SO
      WRITE (6,9)
 9    FORMAT (' ENGLISH UNITS')
      GO TO 12
 10   G=9.806
      GAM=9802.
      CON=RN**2/SO
      WRITE (6,11)
 11   FORMAT (' SI UNITS')
 12   WRITE (6,13) XL,Q,B,Z,RN,SO
 13   FORMAT (/' CHANNEL LENGTH=',F10.1,' DISCHARGE =',F12.3/
     2' B=',F8.2,' Z=',F6.3,' RN=',F6.4,' SO=',F9.6)
      NN=30
C DETERMINATION OF CRITICAL AND NORMAL DEPTHS
      UP=30.
      DN=0.
      YC=15.
      DO 20 I=1,15
      IF (YCRIT(YC)) 14,21,15
 14   DN=YC
      GO TO 20
 15   UP=YC
 20   YC=(UP+DN)*.5
 21   IF (YCONT.EQ.0.) YCONT=YC
      IF (SO.LE.0.) GO TO 33
      UP=40.
      DN=0.
      YN=20.
      DO 30 I=1,15
      IF (YNORM(YN)) 23,31,24
 23   DN=YN
      GO TO 30
 24   UP=YN
 30   YN=(UP+DN)*.5
 31   WRITE (6,32) YN,YC
 32   FORMAT (/' NORMAL DEPTH=',F7.3,' CRITICAL DEPTH=',F7.3)
      GO TO 35
 33   YN=3.*YC
 34   FORMAT (/' CRITICAL DEPTH=',F7.3)
      WRITE (6,34) YC
```

- **The program begins with several line functions to compute the various variables and functions in the problem. After the necessary data input, critical depth is computed, followed by the normal-depth calculation if normal depth exists.**

- **The bisection method is used in these calculations. The type of profile is then categorized, and finally the water-surface profile, specific energy, and *F* + *M* are calculated and printed. Simpson's rule is used in the integration for the water-surface profile.**

- **The program can be used for other channel sections, such as circular or parabolic, by simply changing the line functions at the beginning.**

Example 10

- A trapezoidal channel, B = 2.5 m, side slope = 0.8, has two bottom slopes. The upstream portion is 200 m long, S_0 = 0.025, and the downstream portion, 600 m long, S_0 = 0.0002, n = 0.012. A discharge of 25 m^3/s enters at critical depth from a reservoir at the upstream end, and at the downstream end of the system the water depth is 2 m. Determine the water-surface profiles throughout the system, including jump location.

Solution

- Three separate sets of data, shown in Fig. 17, are needed to obtain the results used to plot the solution as shown in Fig. 18.

- The first set for the steep upstream channel has a control depth equal to zero since it will be automatically assumed critical depth in the program.

- The second set is for the supercritical flow in the mild channel. It begins at a control depth equal to the end depth from the upstream channel and computes the water surface downstream to the critical depth.

- The third set of data uses the 2-m downstream depth as the control depth and computes in the upstream direction. Figure 19 shows the computer output from the last two data sets.

- The jump is located by finding the position of equal *F* + *M* from the output of the last two data sets.

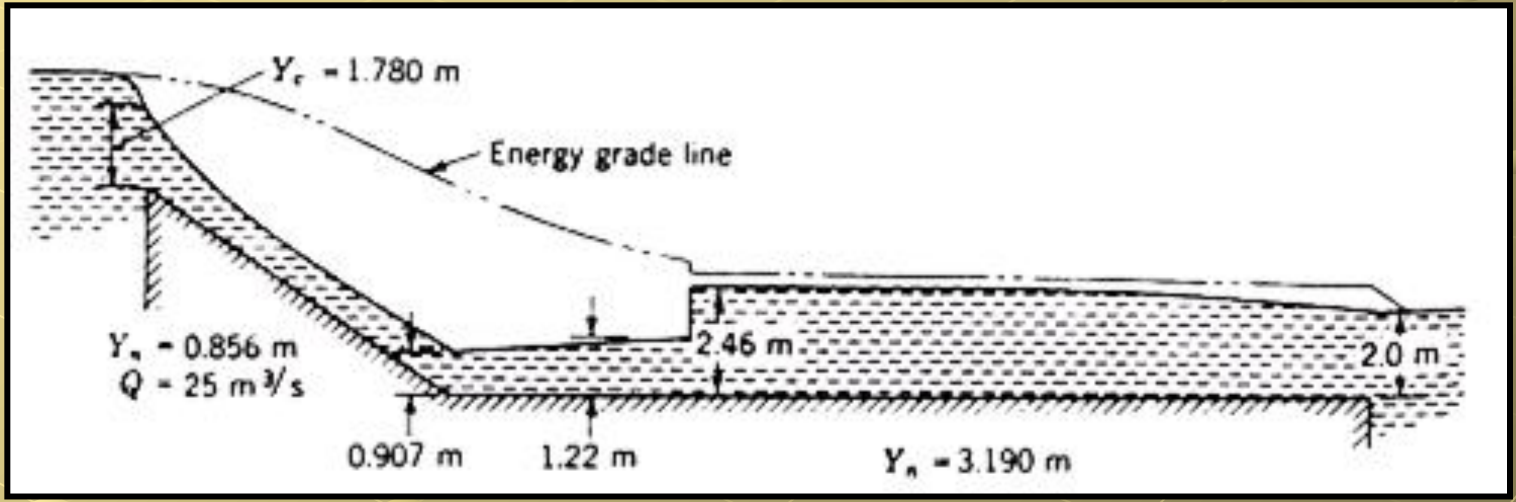


Figure 18 Solution to Example 10 as obtained from computer

Figure 12.19

Computer output

SI UNITS
CHANNEL LENGTH= 600.0 DISCHARGE = 25.000
B= 2.50 Z= 0.800 RN=0.0120 SO= 0.000200
NORMAL DEPTH= 3.190 CRITICAL DEPTH= 1.780
CONTROL IS UPSTREAM, DEPTH= 0.907

DISTANCE	DEPTH	ENERGY	F+M
0.0	0.907	4.630	225573.
25.8	0.965	4.160	211572.
51.2	1.023	3.786	199573.
76.0	1.082	3.487	189272.
100.1	1.140	3.247	180431.
123.3	1.198	3.054	172859.
145.5	1.256	2.900	166399.
166.5	1.315	2.777	160926.
186.0	1.373	2.679	156334.
204.0	1.431	2.603	152534.
220.0	1.489	2.544	149455.
233.9	1.547	2.500	147035.
245.4	1.606	2.469	145221.
254.1	1.664	2.448	143970.
259.6	1.722	2.437	143242.
261.5	1.780	2.433	143007.

SI UNITS
CHANNEL LENGTH= 600.0 DISCHARGE = 25.000
B= 2.50 Z= 0.800 RN=0.0120 SO= 0.000200
NORMAL DEPTH= 3.190 CRITICAL DEPTH= 1.780
CONTROL IS DOWNSTREAM, DEPTH= 2.000

DISTANCE	DEPTH	ENERGY	F+M
0.0	2.000	2.474	146109.
33.0	2.079	2.504	148634.
80.5	2.158	2.541	151844.
145.4	2.238	2.583	155711.
231.3	2.317	2.630	160211.
343.0	2.396	2.681	165326.
486.6	2.475	2.734	171040.
600.0	2.524	2.769	174857.

12.11 FRICTIONLESS POSITIVE SURGE WAVE IN A RECTANGULAR CHANNEL

- In this section the surge wave resulting from a sudden change in flow (due to a gate or other mechanism) that increases the depth is studied. A rectangular channel is assumed, and friction is neglected.
- Such a situation is shown in Fig. 12.20 shortly after a sudden, partial closure of a gate. The problem is analyzed by reducing it to a steady-state problem, as in Fig. 12.21.
- The continuity equation yields, per unit width,

$$(V_1 + c)y_1 = (V_2 + c)y_2 \tag{12.11.1}$$

- and the momentum equation for the control volume 1 - 2, neglecting shear stress on the floor, per unit width, is

$$\frac{\gamma}{2}(y_1^2 - y_2^2) = \frac{\gamma}{g} y_1(V_1 + c)(V_2 + c - V_1 - c) \tag{12.11.2}$$

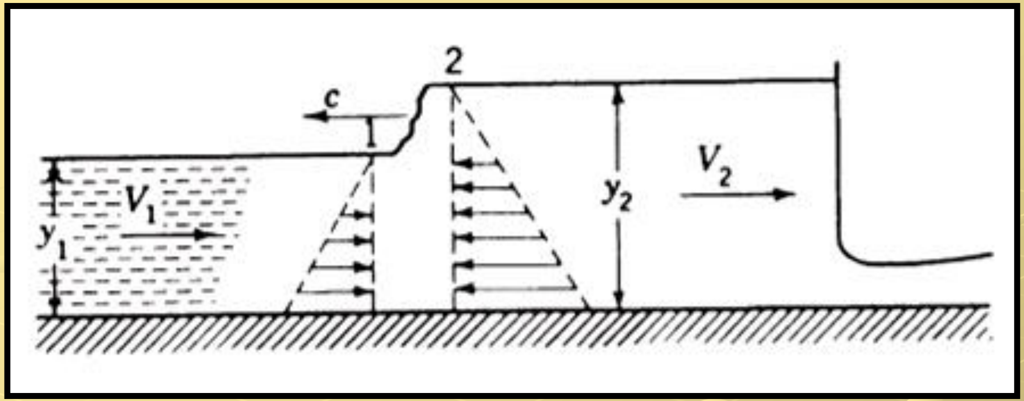
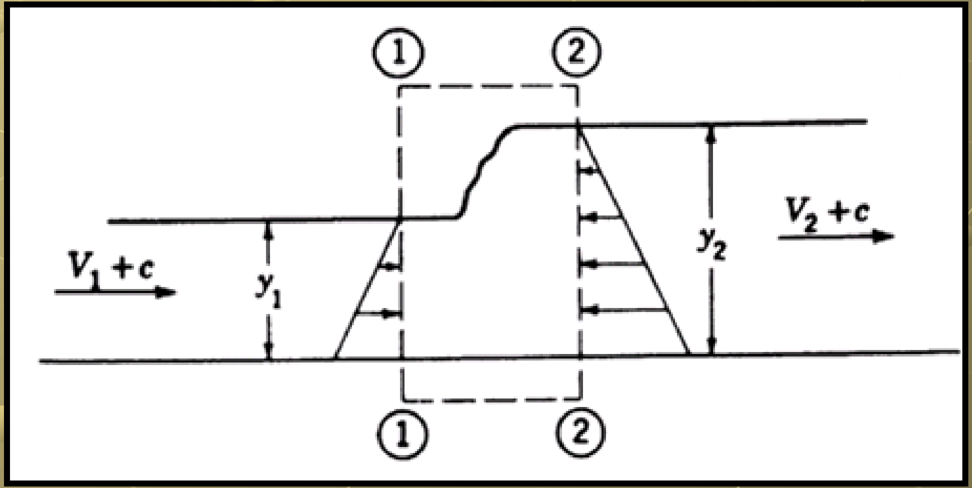


Figure 12.20 Positive surge wave in a rectangular channel

Figure 12.21 Surge problem reduced to a steady-state problem by superposition of surge velocity



- **By elimination of V_2 in the last two equations,**

$$\boxed{V_1 + c = \sqrt{gy_1}\left[\frac{y_2}{2y_1}\left(1 + \frac{y_2}{y_1}\right)\right]^{1/2}} \qquad \textbf{(11.3)}$$

- **In this form the speed of an elementary wave is obtained by letting y_2 approach y_1, yielding**

$$\boxed{V_1 + c = \sqrt{gy}} \qquad \textbf{(11.4)}$$

- **For propagation through still liquid $V_1 \to 0$, and the wave speed is $c = \sqrt{(gy)}$ when the problem is converted back to the unsteady form by superposition of $V = -c$.**

- **In general, Eqs. (11.1) and (11.2) have to be solved by trial. The hydraulic-jump formula results from setting $c = 0$ in the two equations [see Eq. 11.3)].**

Example 11

- A rectangular channel 3 m wide and 2 m deep, discharging 18 m^3/s, suddenly has the discharge reduced to 12 m^3/s at the downstream end. Compute the height and speed of the surge wave.

- Solution $\boxed{V_1 = 3,\ y_1 = 2,\ V_2 y_2 = 4.}$ With Eqs. (11.1) and (11.2),

$$\boxed{6 = 4 + c(y_2 - 2) \qquad \text{and} \qquad y_2^2 - 4 = \frac{2 \times 2}{9.806}(c + 3)(3 - V_2)}$$

- Eliminating *c* and V_2 gives

$$\boxed{\left(\frac{y_2 - 2}{3y_2 - 4}\right)^2 (y_2 + 2)y_2 = \frac{4}{9.806} = 0.407} \qquad \boxed{y_2^2 - 4 = \frac{4}{9.806}\left(\frac{2}{y_2 - 2} + 3\right)\left(3 - \frac{4}{y_2}\right)}$$

- After solving for y_2 by trial, $y_2 = 2.75$ m. Hence, $V_2 = 4/2.75 = 1.455$ m/s, The height of surge wave is 0.75 m, and the speed of the wave is

$$\boxed{c = \frac{2}{y_2 - 2} = \frac{2}{0.75} = 2.667 \text{ m/s}}$$

12 FRICTIONLESS NEGATIVE SURGE WAVE IN A RECTANGULAR CHANNEL

- **The negative surge wave appears as a gradual flattening and lowering of a liquid surface. It occurs, for example, in a channel downstream from a gate that is being closed or upstream from a gate that is being opened. Its propagation is accomplished by a series of elementary negative waves superposed on the existing velocity, each wave traveling at less speed than the one at next greater depth.**

- **Application of the momentum equation and the continuity equation to a small depth change produces simple differential expressions relating wave speed *c*, velocity *V*, and depth *y*.**

- **Integration of the equations yields liquid-surface profile as a function of time, and velocity as a function of depth or as a function of position along the channel and time (*x* and *t*). The fluid is assumed to be frictionless, and vertical accelerations are neglected.**

- In Fig. 22*a* an elementary disturbance is indicated in which the flow upstream has been slightly reduced. For application of the momentum and continuity equations it is convenient to reduce the motion to a steady one, as in Fig. 22*b*, by imposing a uniform velocity *c* to the left.

- The continuity equation is

$$\boxed{(V - \delta V - c)(y - \delta y) = (V - c)y}$$

or, by neglecting the product of small quantities,

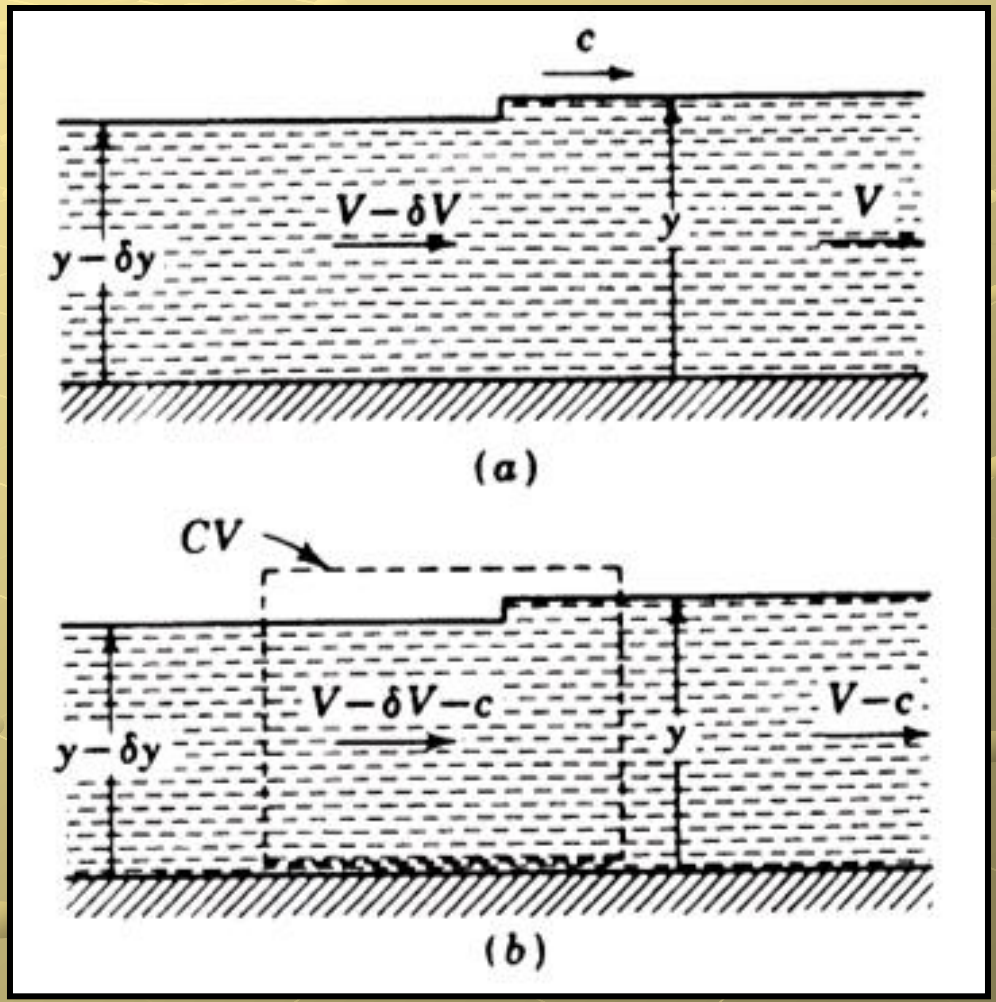
(12.1)

$$\boxed{(c - V)\,\delta y = y\,\delta V}$$

- The momentum equation produces

$$\boxed{\frac{\gamma}{2}(y - \delta y)^2 - \frac{\gamma}{2}y^2 = \frac{\gamma}{g}(V - c)y[V - c - (V - \delta V - c)]}$$

Figure 12.22 Elementary wave



- **After simplifying,**

$$\delta y = \frac{c - V}{g} \delta V \qquad \textbf{(12.12.2)}$$

- **Equating δ*V*/δ*y* in Eqs. (12.12.1) and (12.12.2) gives**

$$\begin{aligned} c - V &= \pm\sqrt{gy} \\ c &= V \pm \sqrt{gy} \end{aligned} \qquad \textbf{(12.12.3)}$$

- **The speed of an elementary wave in still liquid at depth *y* is √(gy) and with flow the wave travels at the speed √(gy) *relative* to the flowing liquid.**
- **Eliminating *c* from Eqs. (12.12.1) and (12.12.2) gives**

$$\frac{dV}{dy} = \pm\sqrt{\frac{g}{y}}$$

- **and integrating leads to**

$$V = \pm 2\sqrt{gy} + \text{const}$$

- **For a negative wave forming downstream form a gate (Fig. 12.23) by using the plus sign, after an instantaneous partial closure, $V = V_0$ when $y = y_0$, and**

$$\boxed{V_0 = 2\sqrt{g y_0} + \text{const}}$$

- **After eliminating the constant,**

$$\boxed{V = V_0 - 2\sqrt{g}\left(\sqrt{y_0} - \sqrt{y}\right)} \qquad (12.12.4)$$

- **The wave travels in the +x direction, so that**

$$\boxed{c = V + \sqrt{gy} = V_0 - 2\sqrt{g y_0} + 3\sqrt{gy}} \qquad (12.12.5)$$

- **If the gate motion occurs at t = 0, the liquid-surface position is expressed by $x = ct$, or**

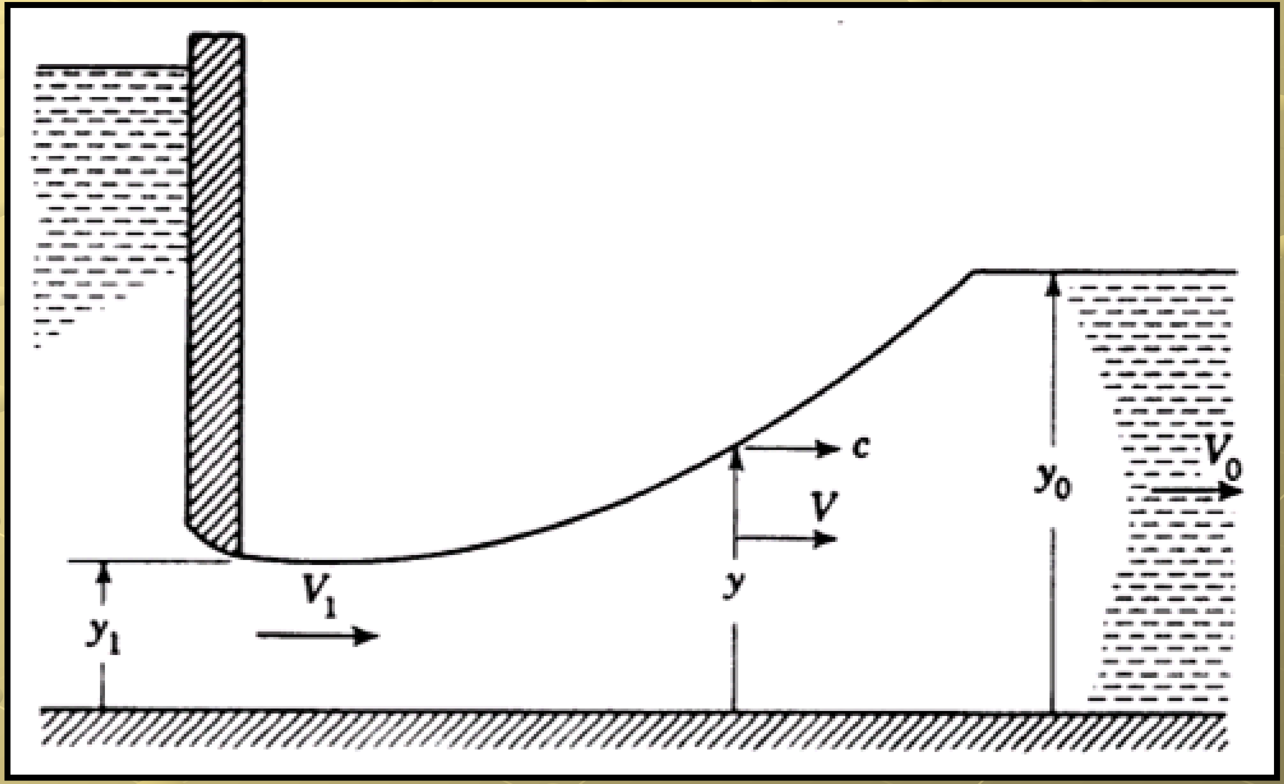
$$\boxed{x = \left(V_0 - 2\sqrt{g y_0} + 3\sqrt{gy}\right)t} \qquad (12.12.6)$$

- **Eliminating y from Eqs. (12.12.5) and (12.12.6) gives**

$$\boxed{V = \frac{V_0}{3} + \frac{2}{3}\frac{x}{t} - \frac{2}{3}\sqrt{g y_0}} \qquad (12.12.7)$$

- **which is the velocity in terms of x and t.**

Figure 23 Negative wave after gate closure



Example 12.12

- In Fig. 12.23 find the Froude number of the undisturbed flow such that the depth y_1, at the gate is just zero when the gate is suddenly closed. For $V_0 = 6$ m/s, find the liquid-surface equation.

Solution

- It is required that $V_1 = 0$ when $y_1 = 0$ at $x = 0$ for any time after $t = 0$. In Eq. (12.12.4), with $V = 0$, $y = 0$,

$$V_0 = 2\sqrt{gy_0} \qquad \text{or} \qquad \mathbf{F}_0 = \frac{V_0}{\sqrt{gy_0}} = 2$$

- For $V_0 = 6$,

$$y_0 = \frac{V_0^2}{4g} = \frac{6^2}{4g} = 0.92 \text{ m}$$

- By Eq. (12.12.6)

$$x = (6 - 2\sqrt{9.806 \times 0.92} + 3\sqrt{9.806y})t = 9.39\sqrt{y}\, t$$

- The liquid surface is a parabola with vertex at the origin and surface concave upward.

Example 12.13

- In Fig. 12.23 the gate is partially closed at the instant $t = 0$ so that the discharge is reduced by 50 percent. $V_0 = 6$ m/s, $y_0 = 3$ m. Find V_1, y_1, and the surface profile.

Solution

- The new discharge is

$$q = \frac{6 \times 3}{2} = 9 = V_1 y_1$$

- By Eq. (12.12.4)

$$V_1 = 6 - 2\sqrt{9.806}\,(\sqrt{3} - \sqrt{y_1})$$

- Then V_1 and y_1 are found by trail from the last two equations, $V_1 = 4.24$ m/s, $y_1 = 2.12$ m. The liquid-surface equation, from Eq. (12.12.6), is

$$x = (6 - 2\sqrt{3g} + 3\sqrt{gy})t \qquad \text{or} \qquad x = (9.39\sqrt{y} - 4.84)t$$

- which holds for the range of values of y between 2.12 and 3 m.

Dam Break

- An idealized dam-break water-surface profile, Fig. 12.24 can be obtained from Eqs. (12.12.4) to (12.12.7). From a frictionless, horizontal channel with depth of water y_0 on one side of a gate and no water on the other side of the gate, the gate is suddenly removed.

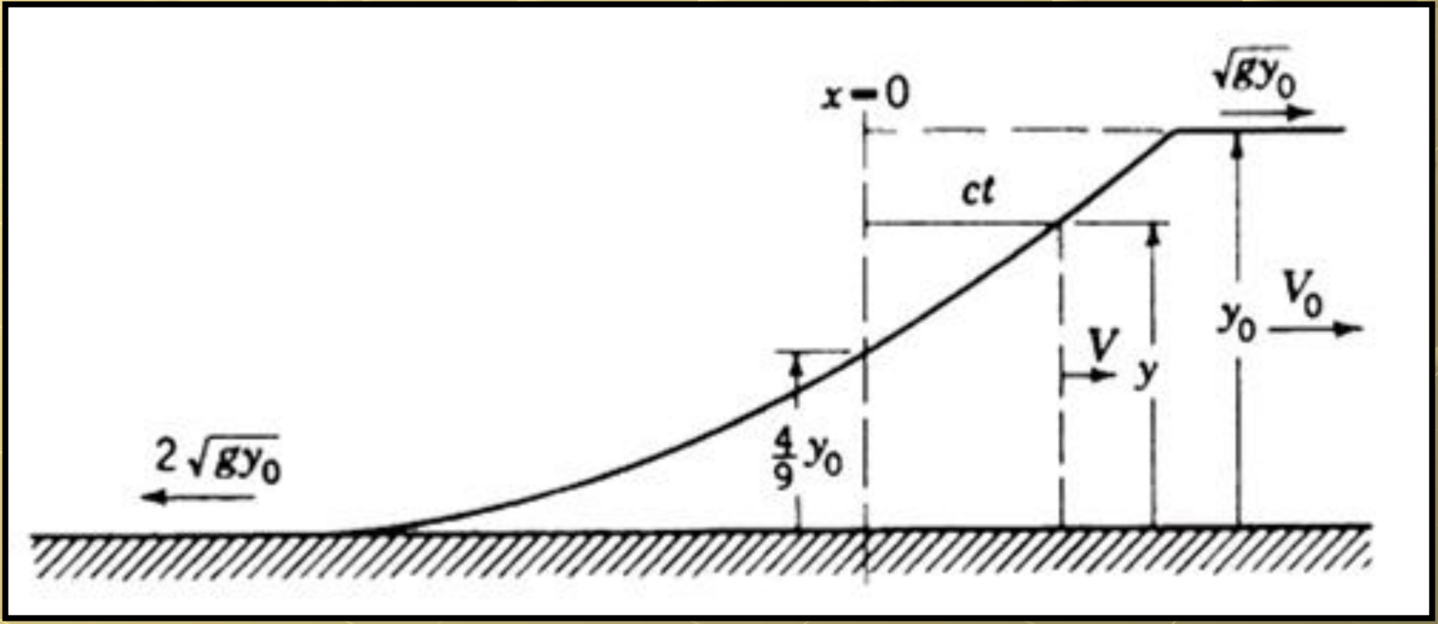
- Vertical accelerations are neglected. $V_0 = 0$ in the equations, and y varies from y_0 to 0. The velocity at any section, from Eq. (12.12.4), is

$$\boxed{V = -2\sqrt{g}\left(\sqrt{y_0} - \sqrt{y}\right)} \qquad (12.12.8)$$

- always in the downstream direction. The water-surface profile is, from Eq. (12.12.6),

$$\boxed{x = \left(3\sqrt{gy} - 2\sqrt{gy_0}\right)t} \qquad (12.12.9)$$

Figure 12.24 Dam-break profile



- **At $x = 0$, $y = 4y_0/9$, the depth remains constant and the velocity past the section $x = 0$ is, from Eq. (12.12.8),**

$$\boxed{V = -\tfrac{2}{3}\sqrt{gy_0}}$$

- **also independent of time.**

- **The leading edge of the wave feathers out to zero height and moves downstream at $V = c = -2\sqrt{(gy_0)}$. The water surface is a parabola with vertex at the leading edge, concave upward.**

- **With an actual dam break, ground roughness causes a positive surge, or wall of water, to move downstream; i.e., the feathered edge is retarded by friction.**